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A chromospheric resonance cavity in a sunspot mapped with seismology
Supplementary Information

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I. PRELUDE

The existence of wave motion is a ubiquitous phenomenon within the confines of sunspot atmospheres[1–4], with pioneering observations dating back to the 1960s providing the first evidence of oscillatory behavior[5,6]. In addition to wave motion being able to act as an energy conduit linking the relatively cool photosphere to the multi-million degree solar corona, the embedded oscillations enable a wealth of important physics to be studied and documented, including:

1. The sub-photospheric drivers responsible for the visible wave motion[7];
2. The phase speed of propagating magneto-hydrodynamic (MHD) waves along the magnetic field lines of a sunspot[8];
3. Interactions between the magnetic field inclination angles and the projected wave phase speeds[9];
4. Seismological estimations of the magnetic field strengths supporting the wave motion[10];
5. The linear and non-linear interactions of upwardly propagating wave modes[11];
6. Estimating the formation height of imaging filters that capture propagating wave dynamics[12];
7. The superposition of multiple wave modes and harmonics[13,14]; and
8. The damping and dissipation of wave energy[15,16].

Hence, for over half a century waves have been used to probe and constrain the plasma characteristics of sunspot atmospheres. However, while sunspot oscillations have been harnessed to provide insight into challenging solar physics questions, the underlying mechanisms describing their evolving composition with atmospheric height has proved elusive.

An interesting characteristic of magnetoacoustic waves manifesting in sunspot atmospheres is that their dominant frequency changes depending upon what geometric height is sampled by the associated observations. At photospheric heights the dominant frequency is on the order of 3 mHz (∼5 min; ref. 17), with the umbral wave power being significantly suppressed when compared to surrounding quiet Sun locations[18]. Many theories explaining the suppression of photospheric umbral wave power have been proposed, including aspects of sub-surface absorption[19], lost energy to the solar interior[20], less efficient generation[21], and the presence of smaller attenuation lengths[22].

Once observations begin to sample increasing geometric heights of the sunspot atmosphere, it is found that the most significant wave power in the chromosphere comes from frequencies around 5 mHz (∼3 min; ref. 23). Due to the mostly vertical nature of the magnetic field lines towards the center of sunspot umbrae, this effect is normally attributed to wave filtering via the acoustic cut-off frequency[24–26], which only allows waves with frequencies higher than ∼5 mHz to propagate upwards. Furthermore, the enhancement of wave power in the chromosphere is striking, with velocity amplitudes rising from a few hundreds of m/s in the photosphere to several km/s in the chromosphere. It must be noted, however, that an increase in velocity amplitude does not necessarily result in an increase in the wave energy flux since the surrounding atmosphere will be significantly less dense[27]. Such a dramatic increase in the velocity amplitudes has been linked to a number of theories. Linking back to the acoustic cut-off frequency, theoretical work[28] has claimed that the prevalence of 3 minute oscillations in the chromosphere was due to the fundamental Lamb effect[29], according to which any disturbance in a stratified atmosphere excites free oscillations at its cut-off frequency. In this scenario, oscillations at approximately 3 minutes already exist in the photosphere, but due to the rapid spatial attenuation of the lower frequency (evanescent) wave components, the 3 minute oscillations begin to dominate the power spectra at chromospheric heights. The Lamb effect, however, is not able to explain the secondary power enhancements found at $\sim$20 mHz in both our observational and numerical time series (see Fig. 2 for more information).

An alternative explanation for the increased wave amplitudes found in chromospheric umbral power spectra comes from the manifestation of powerful shock fronts in the form of umbral flashes[30–35]. Here, magnetoacoustic shocks can manifest across a wide range of atmospheric heights, spanning from the upper photosphere through to the lower transition region[16], and one-dimensional simulations have shown that enhanced 3 minute chromospheric wave power may arise from the wakes of developing shock fronts[36,37]. Observations of such phenomena are typically characterized by 'sawtooth' signatures of the associated spectral profiles as the shock front develops and subsequently cools[35,38]. However, subsequent work[39,40] has revealed that one-dimensional simulations of propagating shock waves lead to unrealistic shock mergers, and as such, full three-dimensional simulations are required to investigate this hypothesis further.

A final explanation for the enhanced umbral wave power found at chromospheric heights comes from the existence of an acoustic resonator[41], which is created due to the substantial temperature gradients experienced at photospheric and transition region heights[42,43]. Such temperature gradients create the semi-permeable boundaries necessary for resonance to occur, whereby partial reflection and transmission takes place as the magnetoacoustic waves encounter the boundary layers[44]. Evidence for both upward and downward propagating MHD waves has been identified in a variety of magnetic structures embodied within the solar atmosphere[45–47]. Ultimately, the resonances produced cause oscillations that leak energy out of the cavity, which should be detectable in observations originating at the base of the transition region and above[48]. Observational studies of MHD wave power throughout the lower solar atmosphere have been conducted[49,50], but were unable to conclusively identify the presence of an embedded resonance cavity. Recent theoretical work[51] numerically investigated the characteristics of a chromospheric resonator using a broadband pulse, which successfully produced the familiar 3 minute chromospheric oscillations. Importantly, further work[40] revealed that the atmosphere (temperature, density, thickness) of the chromospheric resonator itself is fundamentally important to the spectral signatures that are emitted through the resonance cavity and, in particular, demonstrated that the gradient (i.e., spectral slope) of the emitted power spectrum is directly correlated with the chromospheric temperature configuration.

In this work, we employ high resolution He I 10830 Å spectropolarimetric observations, which are formed at the base of the transition region, to undertake sunspot umbral seismology. We propose to reverse previous theoretical work[40] and utilize measured spectral signatures to uncover physical information regarding the structure of the underlying sunspot resonance cavity. Of particular note is the use of He I 10830 Å observations, which are of paramount importance for upcoming next-generation solar telescopes, including the Daniel K. Inouye Solar Telescope[52–54] (DKIST, formerly the Advanced Technology Solar Telescope, ATST), the European Solar Telescope[55] (EST) and the Indian National Large Solar Telescope[56] (NLST). He I 10830 Å observations are unique in the sense that they are magnetically sensitive and the formation layer producing their spectroscopic signatures is often considered optically thin[57], which makes them suitable for very fast inversion routines, such as the HAnle and Zeeman Light[58] (HAZEL) and He-Line Information Extractor[59,60] (HeLIx$^+$) codes. Furthermore, being formed in the infrared means that He I 10830 Å observations are less prone to seeing fluctuations from ground-based observatories, as well as exhibiting increased magnetic field sensitivities when compared to optical chromospheric spectropolarimetric signatures.

II. OVERVIEW

We have presented high resolution spectropolarimetric observations of a sunspot atmosphere captured in the Si I 10827 Å and He I 10830 Å spectral lines. Fourier spectral energy plots of the He I 10830 Å umbral Doppler-velocity time series revealed variable spectral slopes within the interval $18 - 27$ mHz. Relatively shallow gradients (-5.4 ± 0.6) were found close to the umbral core, while steeper slopes (-7.8 ± 0.6) were linked to regions close to the umbra/penumbra boundary.

To further investigate the underlying physical cause of this phenomenon, Lare2D numerical models were employed that made use of a scaled model[61] 'M' atmosphere. A variety of magnetoacoustic wave propagation distances, spanning $1700 - 2545$ km, were utilized to create synthetic Doppler-velocity time series. It was found that the spectral slope of the numerically derived spectral energies (in the range of $18 - 27$ mHz) was directly related to the stratification of the embedded chromospheric resonance cavity. A comparison between the observed spectral gradients and those calculated from the Lare2D models, including consideration of the inclination angles of the magnetic waveguides, revealed geometric heights of the upper chromospheric boundary in the range of 2300 ± 250 km and 1300 ± 200 km for the umbral cores and umbra/penumbra boundaries, respectively.

Such structuring has important implications for sunspot atmospheric seismology, since the umbral atmosphere can no longer be considered as a homogeneous slab environment. Instead, thicknesses of the chromospheric resonance layer will need to be incorporated into seismological estimations in order to improve the accuracy of such techniques. This work also forms new science goals for high-resolution solar physics, notably: (1) *Do all sunspots show similar resonant behavior?* (2) *Do the resonant frequencies/slopes vary in a consistent manner depending on the position of the umbra sampled (i.e., is the resonance cavity always thickest at the core of the umbra)?* (3) *Does the McIntosh/Hale classification and/or evolutionary stage of the sunspot effect the depth* *of the chromospheric resonance cavity?* and (4) *Can spatially-mapped (i.e., two-dimensional) power spectra provide accurate three-dimensional models of sunspot atmospheres?*

With regards to points (1–2) above, we find the cavity-induced resonant peak at $\sim$20 mHz. However, is this frequency universal across all sunspot umbrae, or does it depend on the atmospheric structuring of the sunspot itself? In the present work the $\sim$20 mHz frequency, at which the resonance signatures peak, is independent of the cavity depth since variations in the embedded temperature stratification (i.e., the 80%, 90%, 110%, and 120% input profiles depicted in Extended Data Fig. 4) do not yield well-defined shifts in the corresponding resonant frequency spectrum. As such, the $\sim$20 mHz resonant peak may be related to the pressure scale height found in the umbral atmosphere. Previous attempts to estimate the pressure scale height in different sunspot umbrae have produced incredibly contradictory values (e.g., $\sim 100 - 1600$ km; ref. 62–64). As a result, the close agreement between our observed and simulated spectral energies (including the resonant peak at $\sim$20 mHz) may be a consequence of embedding an accurate atmospheric model derived through use of spectropolarimetric inversions. Furthermore, theoretical work has suggested that varying umbral temperatures may lead to different peaks in the corresponding energy spectrum of the resonating chromospheric cavity[51]. Hence, the absolute umbral temperatures, which correspond to the boundaries of the resonance cavity, may have implications for the frequency corresponding to the spectral peak. Presently, it is unknown what underlying physics produces the distinct spectral energy enhancement at $\sim$20 mHz, even though we have shown that it is a consequence of a stable chromospheric resonance cavity (Extended Data Fig. 6). As such, future research (e.g., with the upcoming DKIST facility) will need to employ high-precision spectropolarimetry to ascertain whether different umbral atmospheric structuring (including the presence of potentially different pressure scale heights) can shift the frequency associated with the resonant peak.

In order to address the science objectives (1–4) listed above, a multi-wavelength statistical study, which includes a vast assortment of sunspot classifications, needs to be undertaken to determine the general resonant properties of sunspot atmospheres. Of course, this is a challenging endeavor, since a wealth of sunspot observations need to be acquired with high spatial and temporal resolutions, complete with high-precision polarimetry of a spectral line formed above the embedded resonance cavity (e.g., He I 10830Å), all while captured during extended periods of good atmospheric seeing conditions.

The He I 10830 Å spectral line is particularly well-suited for this task, since the spectral energy slopes calculated will be relatively insensitive to opacity effects. This means that the spectral gradients are predominantly effected by the resonance cavity size and the embedded thermal structure, hence relatively independent of the precise formation height of the line, which we find to vary from the center to the edge of the umbra. The results documented here are also significant for upcoming DKIST, EST and NLST operations. Simultaneous observations of the Si I 10827 Å and He I 10830 Å spectral lines will be common for these upcoming facilities, with encouragement coming from the fact that the upcoming DKIST, EST and NLST facilities will have instrumentation capable of obtaining the high spatial, spectral and temporal resolutions necessary to tackle this challenging task. In particular, integral field units such as the Diffraction Limited Near Infrared Spectropolarimeter (DL-NIRSP) instrument on DKIST will allow the thicknesses of umbral resonance cavities to be mapped from two-dimensional spectral image sequences, hence providing the unique ability to perform three-dimensional sunspot atmosphere reconstructions from a relatively simplistic two-dimensional spectral power map.

Importantly, wave interactions with resonance cavities are not just limited to the field of solar physics. Other astrophysical research areas, including those linked to energy transfer in magnetars[65], the search for dark-matter axions[66], modeling of Alfvén resonances in the Earth's ionosphere[67–69], the study of quasi-periodicities arising from supermassive binary black hole mergers[70], the examination of strange quark matter associated with white dwarf stars[71], and the quantification of stellar mass loss rates[72], all rely heavily on an accurate understanding of wave physics in a resonance cavity environment. As a result, understanding the physics responsible for the creation of resonance cavities, along with their impact on the universe around us, is of paramount importance. Our present work enhances the astrophysical community's knowledge on what atmospheric characteristics are required to form a stable resonance cavity (e.g., specific temperature stratifications), what impact this has on waveforms interacting with the cavity structure (e.g., power enhancements at well-defined frequencies), and how cutting-edge numerical simulations can be employed alongside high-precision spectropolarimetric data products to deduce physical parameters corresponding to the local plasma conditions (e.g., cavity depth).

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